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Letter to the Editor

On laterally vibrating beams carrying tip masses, coupled by several double spring–mass systems

M. Gürgöze*, H. Erol

Faculty of Mechanical Engineering, Technical University of Istanbul, 80191 Gümüşsuyu, Istanbul, Turkey

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1. Introduction

The present work is concerned with the same mechanical system investigated in Ref. [1]. It is made up of two clamped-free Bernoulli–Euler beams carrying tip masses to which several spring–mass systems are attached across the span. The special case of a symmetric system can be viewed as a model of a suspension bridge tower for studying its bending vibrations, among other applications. In Ref. [1], the Green function method is employed to derive the frequency equation of the system described. The eigenfrequencies are obtained one by one through the numerical solution of a determinantal equation. As also reported in Ref. [2], one encounters sometimes with numerical difficulties in finding the roots of a determinantal transcendent equation which can be very time consuming. On the other hand, it is not obvious which combinations of the physical parameter values would cause such a situation is not known a priori. Motivated by this experience, here an alternative method is given for obtaining the eigenfrequencies of the system above. Although it is acknowledged that the method used follows the classical line, it is the belief of the authors that it enables a design engineer who deals with similar systems, to obtain very accurate approximate values of the eigenfrequencies simultaneously and quickly.

After application of the assumed modes method to the continuous parts of the system, the system is discretized. Then the Lagrange equations formulation is applied, where the displacements of the attachment points of the springs of the double spring–mass systems to the beams are expressed in terms of the generalized co-ordinates. Finally, a generalized eigenvalue problem is formulated through the solution of which the eigenfrequencies of the system can be obtained approximately. In comparison to the Green function method where the eigenfrequencies are obtained one by one through the numerical solution of a determinantal transcendent equation, in the present method, eigenfrequencies can be determined simultaneously without being faced with any numerical problems.

*Corresponding author. Fax: +90-212-245-07-95.
E-mail address: gurgozem@itu.edu.tr (M. Gürgöze).

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2. Theory

The problem to be dealt with in the present study is the natural vibration problem of the system shown in Fig. 1, i.e., a laterally vibrating system consisting of two clamped-free Bernoulli–Euler beams carrying tip mass to which $\bar{n}$ double spring–mass systems (secondary systems) are attached across the span.

The main subject of this study is the derivation of the frequency equation of the system described above. The frequency equation follows directly from the formulation of the Lagrange's equations where the displacements of the attachment points of the secondary systems and those of the tip masses to both beams are expressed in terms of the generalized co-ordinates [2]. The formulation leads to a generalized eigenvalue problem, the solutions of which gives the eigenvalues and hence the eigenfrequencies of the system.The kinetic and potential energies of the system are

$$T = \tfrac{1}{2} m_1 \int_0^{L_1} \dot{w}_1^2(x,t)\,\mathrm{d}x + \tfrac{1}{2} m_2 \int_0^{L_2} \dot{w}_2^2(x,t)\,\mathrm{d}x + \tfrac{1}{2} M_1 \dot{z}_{M_1}^2 + \tfrac{1}{2} M_2 \dot{z}_{M_2}^2 + \tfrac{1}{2} \sum_{j=1}^{\bar{n}} M_{S_j} \dot{z}_j^2, \tag{1}$$

$$V = \tfrac{1}{2} E_1 I_1 \int_0^{L_1} w_1''^2(x,t)\,\mathrm{d}x + \tfrac{1}{2} E_2 I_2 \int_0^{L_2} w_2''^2(x,t)\,\mathrm{d}x + \tfrac{1}{2} \sum_{j=1}^{\bar{n}} k_{1,j}(z_j - z_{01,j})^2 + \tfrac{1}{2} \sum_{j=1}^{\bar{n}} k_{2,j}(z_{02,j} - z_j)^2, \tag{2}$$

where dots and primes denote partial derivatives with respect to time t and the position co-ordinate x, respectively. L_i, m_i and $E_i I_i$ denote the length, the mass per unit length and the

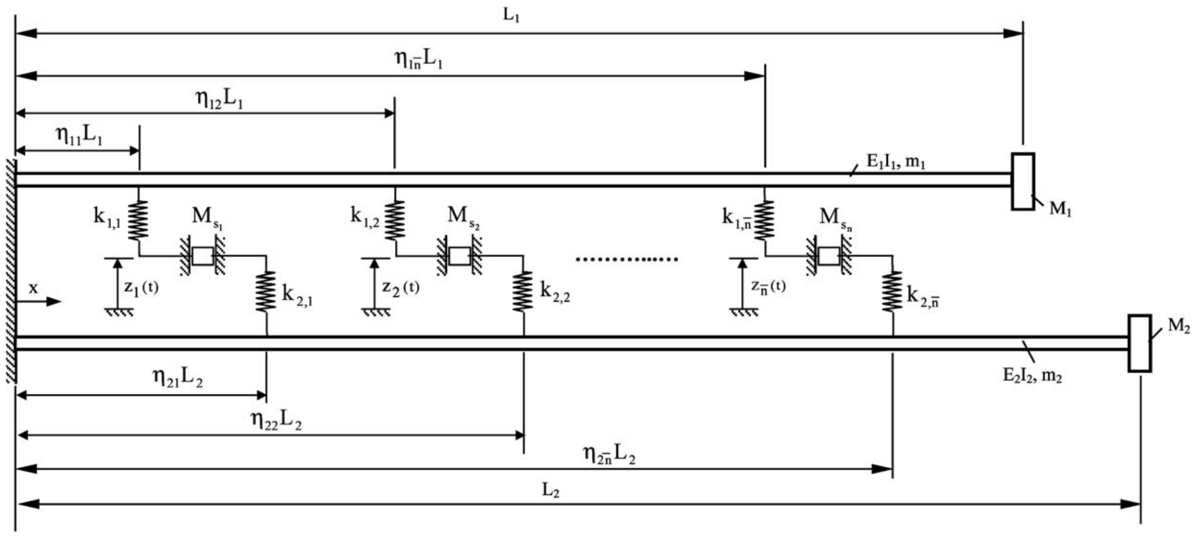

Fig. 1. Two clamped-free laterally vibrating beams carrying tip masses to which $\bar{n}$ double spring–mass systems are attached across the span.

lateral rigidity of the ith beam, respectively ($i = 1, 2$). Further, the bending displacements of the beams are denoted as $w_i(x, t)$. The jth secondary system attached consists of two springs of stiffness's $k_{1,j}$, $k_{2,j}$ and the secondary mass M_{S_j}. The attachment points of this system to the beams are denoted by $\eta_{1j}L_1$ and $\eta_{2j}L_2$, respectively, as shown in Fig. 1. Here, $z_{01,j}$ and $z_{02,j}$ denote the lateral displacements of the attachment points of the jth spring–mass to the first and second beams, respectively, while z_j represents the displacement of the secondary mass M_{S_j}. The lateral displacements of the beams at point x are assumed to be expressible in the form of finite series

$$w_1(x, t) = \sum_{i=1}^{n} W_{i1}(x)\eta_{i1}(t),$$

$$w_2(x, t) = \sum_{i=1}^{n} W_{i2}(x)\eta_{i2}(t), \tag{3}$$

where

$$W_{i1}(x) = \frac{1}{\sqrt{m_1 L_1}}[\cosh \beta_i x - \cos \beta_i x - \bar{\eta}_i(\sinh \beta_i x - \sin \beta_i x)],$$

$$W_{i2}(x) = \frac{1}{\sqrt{m_2 L_2}}[\cosh \beta_i x - \cos \beta_i x - \bar{\bar{\eta}}_i(\sinh \beta_i x - \sin \beta_i x)],$$

$$\bar{\eta}_i = \frac{\cosh \beta_i L_1 + \cos \beta_i L_1}{\sinh \beta_i L_1 + \sin \beta_i L_1}, \quad \bar{\bar{\eta}}_i = \frac{\cosh \beta_i L_2 + \cos \beta_i L_2}{\sinh \beta_i L_2 + \sin \beta_i L_2} \tag{4}$$

are the mass orthonormalized eigenfunctions of both clamped-free Bernoulli–Euler beams and $\eta_{i1}(t)$ and $\eta_{i2}(t)$ ($i = 1, \ldots, n$) are generalized co-ordinates to be determined.

If the assumed series solutions Eq. (3) are substituted into the energy Eqs. (1) and (2), they can be expressed as

$$T = \tfrac{1}{2}\sum_{i=1}^{n} \dot{\eta}_{i1}^2 + \tfrac{1}{2}\sum_{i=1}^{n} \dot{\eta}_{i2}^2 + \tfrac{1}{2} M_1 \dot{z}_{M_1}^2 + \tfrac{1}{2} M_2 \dot{z}_{M_2}^2 + \tfrac{1}{2}\sum_{j=1}^{\bar{n}} M_{S_j} \dot{z}_j^2, \tag{5}$$

$$V = \tfrac{1}{2}\sum_{i=1}^{n} \omega_{i1}^2 \eta_{i1}^2 + \tfrac{1}{2}\sum_{i=1}^{n} \omega_{i2}^2 \eta_{i2}^2 + \tfrac{1}{2}\sum_{j=1}^{\bar{n}} k_{1,j}(z_j - z_{01,j})^2 + \tfrac{1}{2}\sum_{j=1}^{\bar{n}} k_{2,j}(z_{02,j} - z_j)^2, \tag{6}$$

where the orthonormalization properties of the eigenfunctions in Eq. (4) are taken into account. Here, ω_{i1} and ω_{i2} denote the ith eigenfrequencies of the bare cantilevered beams in Fig. 1. Using matrix notations the energy expressions in Eqs. (5) and (6) can further be expressed as

$$T = \tfrac{1}{2}\dot{\boldsymbol{\eta}}^{\mathrm{T}} \mathbf{I}_{2n} \dot{\boldsymbol{\eta}} + \tfrac{1}{2} M_1 \dot{z}_{M_1}^2 + \tfrac{1}{2} M_2 \dot{z}_{M_2}^2 + \tfrac{1}{2}\sum_{j=1}^{\bar{n}} M_{S_j} \dot{z}_j^2, \tag{7}$$

$$V = \tfrac{1}{2}\boldsymbol{\eta}^{\mathrm{T}} \boldsymbol{\Omega}^2 \boldsymbol{\eta} + \tfrac{1}{2}\sum_{j=1}^{\bar{n}} k_{1,j}(z_j - z_{01,j})^2 + \tfrac{1}{2}\sum_{j=1}^{\bar{n}} k_{2,j}(z_{02,j} - z_j)^2, \tag{8}$$

where

$$\boldsymbol{\eta}_1^{\mathrm{T}}(t) = [\eta_{11}(t), \ldots, \eta_{n1}(t)], \quad \boldsymbol{\eta}_2^{\mathrm{T}}(t) = [\eta_{12}(t), \ldots, \eta_{n2}(t)],$$
$$\boldsymbol{\eta}^{\mathrm{T}}(t) = [\boldsymbol{\eta}_1^{\mathrm{T}}, \boldsymbol{\eta}_2^{\mathrm{T}}], \quad \boldsymbol{\Omega}_1^2 = \mathrm{diag}(\omega_{i1}^2), \quad \boldsymbol{\Omega}_2^2 = \mathrm{diag}(\omega_{i2}^2),$$
$$\boldsymbol{\Omega}_2^2 = \mathrm{diag}(\boldsymbol{\Omega}_1^2, \boldsymbol{\Omega}_2^2) \quad (i = 1, \ldots, n), \tag{9}$$

and $\mathbf{I}_{2n}$ denotes the $(2n \times 2n)$ identity matrix.

The idea behind this approach is to express the displacements of the spring attachment points and point masses on to the beams, i.e., $z_{01,j}(t)$, $z_{02,j}(t)$, $z_{M_1}(t)$ and $z_{M_2}(t)$ $(j = 1, \ldots, \bar{n})$ in terms of the vectors $\boldsymbol{\eta}_1(t)$, $\boldsymbol{\eta}_2(t)$ and hence of $\boldsymbol{\eta}(t)$

$$z_{01,j}(t) = w_1(\eta_{1j}L_1, t) = \mathbf{W}_1^{\mathrm{T}}(\eta_{1j}L_1)\boldsymbol{\eta}_1(t) = \boldsymbol{l}_{1j}^{\mathrm{T}}\boldsymbol{\eta}(t),$$
$$z_{02,j}(t) = w_2(\eta_{2j}L_2, t) = \mathbf{W}_2^{\mathrm{T}}(\eta_{2j}L_2)\boldsymbol{\eta}_2(t) = \boldsymbol{l}_{2j}^{\mathrm{T}}\boldsymbol{\eta}(t),$$
$$z_{M_1}(t) = w_1(L_1, t) = \mathbf{W}_1^{\mathrm{T}}(L_1)\boldsymbol{\eta}_1(t),$$
$$z_{M_2}(t) = w_2(L_2, t) = \mathbf{W}_2^{\mathrm{T}}(L_2)\boldsymbol{\eta}_2(t) \tag{10}$$

with

$$\mathbf{W}_1^{\mathrm{T}}(x) = [W_{11}(x), \ldots, W_{n1}(x)], \quad \mathbf{W}_2^{\mathrm{T}}(x) = [W_{12}(x), \ldots, W_{n2}(x)],$$
$$\boldsymbol{l}_{1j}^{\mathrm{T}} = [W_{11}(\eta_{1j}L_1), \ldots, W_{n1}(\eta_{1j}L_1); 0, \ldots, 0],$$
$$\boldsymbol{l}_{2j}^{\mathrm{T}} = [0, \ldots, 0; W_{12}(\eta_{2j}L_2), \ldots, W_{n2}(\eta_{2j}L_2)]. \tag{11}$$

The vectors $\boldsymbol{l}_{1j}$ and $\boldsymbol{l}_{2j}$ are $2n \times 1$ vectors.

Starting with the energy expressions Eqs. (7) and (8), along with Eqs. (9)–(11) the following matrix differential equation is obtained, by using the Lagrange's formulation:

$$\left[\begin{array}{cc|c} \mathbf{I}_n + \alpha_{M_1}\bar{\mathbf{W}}_1(L_1)\bar{\mathbf{W}}_1^{\mathrm{T}}(L_1) & \mathbf{0}_{n\times n} & \\ \hline \mathbf{0}_{n\times n} & \mathbf{I}_n + \alpha_{M_2}\bar{\mathbf{W}}_2(L_2)\bar{\mathbf{W}}_2^{\mathrm{T}}(L_2) & \mathbf{0}_{2n\times\bar{n}} \\ \hline \multicolumn{2}{c|}{\mathbf{0}_{\bar{n}\times 2n}} & \mathbf{M}_S \end{array}\right] \begin{bmatrix} \ddot{\boldsymbol{\eta}} \\ \ddot{\mathbf{z}} \end{bmatrix}$$
$$+ \left[\begin{array}{c|c} \boldsymbol{\Omega}^2 + \boldsymbol{l}_1\mathbf{k}_1\boldsymbol{l}_1^{\mathrm{T}} + \boldsymbol{l}_2\mathbf{k}_2\boldsymbol{l}_2^{\mathrm{T}} & -(\boldsymbol{l}_1\mathbf{k}_1 + \boldsymbol{l}_2\mathbf{k}_2) \\ \hline -(\boldsymbol{l}_1\mathbf{k}_1 + \boldsymbol{l}_2\mathbf{k}_2)^{\mathrm{T}} & \mathbf{k}_1 + \mathbf{k}_2 \end{array}\right] \begin{bmatrix} \boldsymbol{\eta} \\ \mathbf{z} \end{bmatrix} = \mathbf{0}, \tag{12}$$

where

$$\mathbf{W}_1(x) = \frac{1}{\sqrt{m_1 L_1}}\overline{\mathbf{W}}_1(x), \quad \mathbf{W}_2(x) = \frac{1}{\sqrt{m_2 L_2}}\overline{\mathbf{W}}_2(x),$$
$$\mathbf{z}^{\mathrm{T}} = [z_1, \ldots, z_{\bar{n}}], \quad \mathbf{M}_S = \mathrm{diag}(M_{Sj}),$$
$$\mathbf{k}_1 = \mathrm{diag}(k_{1,j}), \quad \mathbf{k}_2 = \mathrm{diag}(k_{2,j}) \quad (j = 1, \ldots, \bar{n}),$$
$$\boldsymbol{l}_1 = [\boldsymbol{l}_{11}, \ldots, \boldsymbol{l}_{1\bar{n}}], \quad \boldsymbol{l}_2 = [\boldsymbol{l}_{21}, \ldots, \boldsymbol{l}_{2\bar{n}}]. \tag{13}$$

It is worth noting that in obtaining the above form of equation of motion, extensive use is made of the formulas regarding the partial derivatives of bilinear forms, quadratic forms and vectors with respect to algebraic vectors [3].

Introducing harmonic solutions of the form

$$\begin{bmatrix} \boldsymbol{\eta} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{\eta}} \\ \bar{\mathbf{z}} \end{bmatrix} e^{i\omega t}. \tag{14}$$

ω being the eigenfrequency of the system, leads to the following generalized eigenvalue problem

$$\mathbf{K}\begin{bmatrix} \bar{\boldsymbol{\eta}} \\ \bar{\mathbf{z}} \end{bmatrix} = \bar{\lambda}\mathbf{M}\begin{bmatrix} \bar{\boldsymbol{\eta}} \\ \bar{\mathbf{z}} \end{bmatrix}, \tag{15}$$

where the $(2n+\bar{n}) \times (2n+\bar{n})$-dimensional matrices $\mathbf{M}$ and $\mathbf{K}$ are as follows:

$$\mathbf{M} = \left[\begin{array}{c:c} \mathbf{I}_{2n} + \alpha_{M_1}\bar{\bar{\mathbf{W}}}_1\bar{\bar{\mathbf{W}}}_1^{\mathrm{T}} + \alpha_{M_2}\bar{\bar{\mathbf{W}}}_2\bar{\bar{\mathbf{W}}}_2^{\mathrm{T}} & \mathbf{0}_{2n\times\bar{n}} \\ \hdashline \mathbf{0}_{\bar{n}\times 2n} & \mathbf{I}_{\bar{n}} \end{array}\right],$$

$$\mathbf{K} = \left[\begin{array}{c:c} \bar{\boldsymbol{\Lambda}} + \bar{\mathbf{e}}_1\boldsymbol{\alpha}_{k_1}\bar{\mathbf{e}}_1^{\mathrm{T}} + \dfrac{\chi}{\alpha_m\alpha_L^4}\bar{\mathbf{e}}_2\boldsymbol{\alpha}_{k_2}\bar{\mathbf{e}}_2^{\mathrm{T}} & -\left(\bar{\mathbf{e}}_1\boldsymbol{\alpha}_{k_1}\boldsymbol{\alpha}_{M_S}^{-1/2} + \dfrac{\chi}{\alpha_L^3\sqrt{\alpha_m\alpha_L}}\bar{\mathbf{e}}_2\boldsymbol{\alpha}_{k_2}\boldsymbol{\alpha}_{M_S}^{-1/2}\right) \\ \hdashline -\left(\bar{\mathbf{e}}_1\boldsymbol{\alpha}_{k_1}\boldsymbol{\alpha}_{M_S}^{-1/2} + \dfrac{\chi}{\alpha_L^3\sqrt{\alpha_m\alpha_L}}\bar{\mathbf{e}}_2\boldsymbol{\alpha}_{k_2}\boldsymbol{\alpha}_{M_S}^{-1/2}\right)^{\mathrm{T}} & \left(\boldsymbol{\alpha}_{k_1} + \dfrac{\chi}{\alpha_L^3}\boldsymbol{\alpha}_{k_2}\right)\boldsymbol{\alpha}_{M_S}^{-1} \end{array}\right]. \tag{16}$$

The following definitions are used in the above expressions:

$$\lambda_i = \bar{\beta}_i^4, \quad \bar{\beta}_i = \beta_i L_1, \quad \bar{\beta}_1 = 1.875104068712, \quad \bar{\beta}_2 = 4.694091132974, \ldots,$$

$$\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_i), \quad \bar{\boldsymbol{\Lambda}} = \mathrm{diag}\left(\boldsymbol{\Lambda}, \frac{1}{\delta^4}\boldsymbol{\Lambda}\right), \quad \bar{\bar{\mathbf{W}}}_1 = \begin{bmatrix} \bar{\mathbf{W}}_1(L_1) \\ \mathbf{0}_{n\times 1} \end{bmatrix}, \quad \bar{\bar{\mathbf{W}}}_2 = \begin{bmatrix} \mathbf{0}_{n\times 1} \\ \bar{\mathbf{W}}_2(L_2) \end{bmatrix},$$

$$\omega_{i1}^2 = \lambda_i\omega_{01}^2, \quad \omega_{i2}^2 = \lambda_i\omega_{02}^2, \quad \omega_{01}^2 = \frac{E_1I_1}{m_1L_1^4}, \quad \omega_{02}^2 = \frac{E_2I_2}{m_2L_2^4} = \frac{1}{\delta^4}\omega_{01}^2,$$

$$\omega^2 = \bar{\beta}^4\omega_{01}^2, \quad \delta^4 = (\mu\alpha_L)^4, \quad \mu^4 = \frac{\alpha_m}{\chi}, \quad \alpha_m = \frac{m_2}{m_1},$$

$$\alpha_L = \frac{L_2}{L_1}, \quad \chi = \frac{E_2I_2}{E_1I_1}, \quad \bar{\lambda} = \bar{\beta}^4,$$

$$\alpha_{M_1} = \frac{M_1}{m_1L_1}, \quad \alpha_{M_2} = \frac{M_2}{m_2L_2}, \quad \alpha_{k_{1,j}} = \frac{k_{1,j}}{E_1I_1/L_1^3}, \quad \alpha_{k_{2,j}} = \frac{k_{2,j}}{E_2I_2/L_2^3}, \quad \alpha_{M_{S_j}} = \frac{M_{S_j}}{m_1L_1},$$

$$\bar{\mathbf{e}}_1 = \sqrt{m_1L_1}\,l_1, \quad \bar{\mathbf{e}}_2 = \sqrt{m_2L_2}\,l_2, \quad \boldsymbol{\alpha}_{k1} = \mathrm{diag}(\alpha_{k1,j}),$$
$$\boldsymbol{\alpha}_{k2} = \mathrm{diag}(\alpha_{k2,j}), \quad \boldsymbol{\alpha}_{M_S} = \mathrm{diag}(\alpha_{M_{S_j}}) \quad (j = 1, \ldots, \bar{n}). \tag{17}$$

The solution of the eigenvalue problem Eq. (15) yields the non-dimensional eigenfrequency parameters $\bar{\beta}$ of the mechanical system in Fig. 1 via $\bar{\beta} = \bar{\lambda}^{1/4}$.

3. Numerical results

This section is devoted to the numerical evaluations of the formulae established in the preceding sections. As the first numerical application, a system with only one secondary system, i.e., $\bar{n} = 1$, is taken. The following values are chosen for the physical data of the mechanical system in Fig. 1. $\eta_{11} = 0.50$, $\eta_{21} = 0.50$, $\alpha_{k_{1,1}} = \alpha_{k_{2,1}} = 1000$, $\alpha_{M_1} = \alpha_{M_2} = 2$, $\alpha_m = 1$, $\alpha_{M_{S_1}} = 1$, $\alpha_L = 1$, $\chi = 1$. It is seen clearly that the system under consideration is a symmetrical one. These numerical values are the values, used also in Ref. [1]. The number of the modes n in Eq. (3) is chosen as 15. The first 10 dimensionless eigenfrequency parameters $\bar{\beta}$ of the system above are collected in Table 1.

The first column contains those $\bar{\beta}$ values taken from Ref. [1] which were obtained on the basis of the Green function method. The figures in the second column represent the fourth roots of the eigenvalues $\bar{\lambda}$ of the solution of the generalized eigenvalue problem formulated in Eq. (15) of the present study. The solution of the eigenvalue problem is performed with MATLAB.

The comparison of the numbers in both columns indicates clearly that the present approach yields very good approximations to the "exact" eigenfrequency parameters in the first column obtained via the Green function method.

As a second example, a sample system with $\bar{n} = 2$, i.e., with two secondary systems is taken. The chosen physical data are as follows: $\eta_{11} = \eta_{21} = 0.50$, $\eta_{12} = \eta_{22} = 0.75$, $\alpha_{k_{1,1}} = \alpha_{k_{1,2}} = 4$, $\alpha_{k_{2,1}} = \alpha_{k_{2,2}} = 6$, $\alpha_{M_1} = \alpha_{M_2} = 7$, $\alpha_m = 1$, $\alpha_{M_{S_1}} = \alpha_{M_{S_2}} = 5$, $\alpha_L = 1$, $\chi = 1$.

The number n in Eq. (3) is taken again as 15. The first 12 dimensionless eigenfrequency parameters $\bar{\beta}$ of the system above are given in Table 2. The first column contains those $\bar{\beta}$ values which are determined by the Green function method given in Ref. [1]. The numbers in the second column represent the fourth roots of the eigenvalues $\bar{\lambda}$ of the solution of the generalized eigenvalue problem in Eq. (15). The comparison of the numbers in both columns reveals a good agreement indicating that the present approach yields very good approximations to the "exact"

Table 1
Dimensionless eigenfrequency parameters $\bar{\beta}$ of the system in Fig. 1 with only one secondary system, i.e., $\bar{n} = 1$

From Ref. [1]	From Eq. (15)
1.070263	1.070263
1.558790	1.558790
3.309592	3.309592
6.334255	6.334282
6.819830	6.819830
7.469383	7.469383
7.925956	7.925956
10.666736	10.666736
10.746971	10.746971
13.402318	13.404682

Table 2
Dimensionless eigenfrequency parameters $\bar{\beta}$ of the system in Fig. 1 with two secondary systems, i.e., $\bar{n} = 2$

Via the method in Ref. [1]	From Eq. (15)
0.764696	0.764693
0.922096	0.922087
1.183296	1.183294
1.231295	1.231287
4.000036	3.999825
4.027484	4.027278
7.084902	7.084860
7.088080	7.087985
10.219316	10.219295
10.220430	10.220384
13.357241	13.357239
13.357319	13.357315

eigenfrequency parameters, in this example too. One could expect that the dimensionless eigenfrequencies $\bar{\beta}$ obtained from Eq. (15) converge to those from the Green function approach as n gets larger, but there is not enough evidence in order to make this statement.

The great advantage of the approach used in the present study is that all eigenfrequency parameters of the system are obtained simultaneously and without any difficulties. On the contrary, by using the Green function method, these parameters have to be found via one by one numerical search of the roots of a determinantal equation which is of transcendental nature. It is a well-known fact that finding the roots of transcendental equations is associated often with numerical difficulties.

4. Conclusions

This paper deals with the eigencharacteristics of a laterally vibrating system made up of two clamped-free Bernoulli–Euler beams carrying tip masses to which several double spring–mass systems are attached across the span. After discretizing via the assumed modes method, the Lagrange's equations formulation is applied, where the displacements of the spring attachment points and those of the tip masses to both beams are expressed in terms of the generalized co-ordinates. This procedure leads to a generalized eigenvalue problem. The eigenvalues of it yield the eigenfrequencies of the system simultaneously. The numerical results obtained reveal that the eigenfrequencies calculated by this method are in good agreement with those obtained by the Green function method.

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References

[1] S. Inceoğlu, M. Gürgöze, Bending vibrations of beams coupled by several spring–mass systems, Journal of Sound and Vibration 243 (2001) 370–379.

[2] M. Gürgöze, On the alternative formulations of the frequency equation of a Bernoulli–Euler beam to which several spring–mass systems are attached in-span, Journal of Sound and Vibration 217 (1998) 585–595.

[3] E.J. Haug, Computer Aided Kinematics and Dynamics of Mechanical Systems I: Basic Methods, Allyn and Bacon, Boston, 1989.